Obstacle Problems in Variable Exponent Sobolev Spaces

We are going to discuss the obstacle problem for a quite large class of heterogeneous quasi-linear degenerate elliptic operators in variable exponent Sobolev spaces (Orlicz-Sobolev spaces). We see that the solution has $C^{1,\alpha}_{loc}$ regularity for some $\alpha \in (0,1)$ and prove that the free boundary is a porous set, and hence has Lebesgue measure zero. For a specific class of operators we prove the finiteness of the $(n-1)$ dimensional Hausdorff measure of the free boundary.

If time allows, a model of a steady glacier flow also will be discussed. We represent the case as an obstacle problem in Orlicz-Sobolev spaces - with the corresponding equation being a fully nonlinear PDE. We show the existence of a solution using a fixed point argument.